Problem 14. Let R be a ring and let $f\colon M \to N$ be an R-morphism. Suppose that $\ker(f)$ and $\mathrm{im}(f)$ are finitely generated. Show that M is finitely generated.

Problem 15. Let R be a ring and M an R-module. An *R-endomorphism* of M is a R-morphism $f\colon M \to M$. Check that the set of all R-endomorphisms, $\mathrm{End}_R(M)$, is a ring with composition as multiplication and pointwise addition. If R is commutative, $\mathrm{End}_R(M)$ is even an R-algebra.

Problem 16. Let R be a ring. An R-module M is called *simple* if $M \neq 0$ (here $0 = \{0\}$ is the zero module), and the only submodules of M are M itself and 0.

- (Schur's Lemma) Let M, N be simple R-modules and $f\colon M \to N$ a R-morphism. Prove that either $f = 0$ or f is an isomorphism.
- Show: if M is simple, then $\mathrm{End}_R(M)$ is a division ring.