We discuss Problem 11(b) and (c).

Problem 12. Let R be a ring. The opposite ring R^{op} is obtained from R by reversing the multiplication: $a \cdot_{\mathrm{op}} b \coloneqq ba$.

(a) Check that R^{op} is a ring.

(b) Show that every left R-module M is a right R^{op}-module, via $mr \coloneqq rm$ for $r \in R$ and $m \in M$.

(c) Show that the identity map is a ring isomorphism $R \cong R^{\mathrm{op}}$ if and only if R is commutative.

(d) Show that $\mathbb{R}_{n,n}$ is isomorphic to its opposite ring for every $n \geq 1$ (not via the identity map for $n \geq 2$).

Problem 13. For a ring A, the center is defined as

$$Z(A) \coloneqq \{\, x \in A : \forall y \in A : xy = yx \,\}.$$

Show:

(a) $Z(A)$ is a ring.

(b) If R is a commutative ring, then the ring A is a (unitary) associative R-algebra if and only if there exists a ring homomorphism $\varphi \colon R \to Z(A)$.