Throughout, let $(a_n)_{n\geq 0}$ be a strict $\mathbb{Q}$-LRS of order d.

Exercise 1. If $(a_n)_{n\geq 0}$ has at least d consecutive zeros, then it is identical to 0.

Exercise 2. The subsequence $(a_{Nn+r})_{n\geq 0}$ is a (strict) LRS for all $N \geq 1$, $r \geq 0$.

Exercise 3. If $(a_n)_{n\geq 0}$ is nonzero but has infinitely many zeros, then it has two eigenvalues $\lambda \neq \lambda' \in \overline{\mathbb{Q}}$ such that $(\lambda'/\lambda)^m = 1$ for some $m \geq 1$. Such an LRS is called *degenerate*.
(Here $\overline{\mathbb{Q}}$ is the algebraic closure of $\mathbb{Q}$; you can also work in $\mathbb{C}$.)

Exercise 4. It is possible to compute $N \in \mathbb{N}$, $k_1, \ldots, k_r \geq 0$ such that each $(a_{Nn+k_i})_{n\geq 0}$ is identically zero, and such that $(a_n)_{n\geq 0}$ has only finitely many other zeros.