Exercise 1. Determine exponential polynomial representations and the generating series of the following LRS (over $\mathbb{C}$):

(a) $a_n = a_{n-2}$ for $n \geq 2$ with $a_0 = 2$, $a_1 = 0$,

(b) $a_n = 4a_{n-1} - 5a_{n-2} + 2a_{n-3}$ for $n \geq 3$, with $a_0 = 3$, $a_1 = 11$, $a_2 = 22$.

Exercise 2. Show that, for all $j \geq 1$, in the formal power series ring $K[\![x]\!]$,

$$\frac{1}{(1-x)^j} = \sum_{n=0}^{\infty} \binom{n+j-1}{j-1} x^n.$$

(Say, for $\operatorname{char} K = 0$, but this also works in positive characteristic $p > 0$ if one interprets $\binom{n+j-1}{j-1}$ as the reduction of the binomial coefficient modulo p.)

Exercise 3. Where is $\operatorname{char} K = 0$ used in Proposition 1.3? What happens if $\operatorname{char} K = p > 0$?

Exercise 4. For an algebraically closed field of characteristic 0 (e.g., $\mathbb{C}$), prove the bijection between strict LRS and exponential polynomials in the final remark.